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Invertible Fuzzy Topological Spaces

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Preface

The introduction of fuzzy sets in the world of mathematics was a paradigm shift, as it represented the uncertainties of real-life systems. Developed by combining the ordered structure and topological properties, fuzzy topology plays a pivotal role in nurturing local as well as global nature of classical topological properties. Investigated and experimented in a fuzzy sense, the fuzzy sets are undoubtedly a revolutionary move.

This book is a detailed study of invertible fuzzy topological spaces. Here, the main attention is paid to inverting pairs in a fuzzy topological space, local to global fuzzy topological properties and the associated spaces of invertible fuzzy topological spaces. Invertible L-topological spaces are also studied here. The book encompasses a diverse learning on invertibility in fuzzy topological spaces and discusses at length the basic concepts of fuzzy topology and L-topology. The chapters are presented in a very systematic way with plethora of examples that the readers can easily comprehend the concept presented in a readable manner with simple notations.

Chapters Description

The book consists of seven chapters. Chapter 1 illustrates the motivation behind the study of invertible fuzzy topological spaces. In this chapter, all the basic definitions and results for the subsequent reading are given. But, it is recommended to the reader to have a basic idea of fuzzy sets and its operations and also basic concepts of classic topology.

Chapter 2 classifies certain fuzzy topological spaces based on homeomorphisms introducing the concept of N-fuzzy topological spaces, strongly homogeneous fuzzy topological spaces, H-fuzzy topological spaces, complete H-fuzzy topological spaces and H-fuzzy topological spaces of degree n. Besides, the relationship between them, their connection with homogeneous and completely homogeneous

fuzzy topological spaces have also been investigated. Finally, the sums, subspaces and simple extensions of these fuzzy topological spaces are explored.

Chapter 3 concentrates on the basic nature of invertible as well as completely invertible fuzzy topological spaces. The role of fuzzy points on the invertibility of a fuzzy topological space is examined. Apart from certain classes of invertible fuzzy topological spaces, conditions for the invertibility of some other types of fuzzy topological spaces are also obtained. In addition, situations under which a given fuzzy set is not an inverting fuzzy set are explored. It is observed that every fuzzy topological space can be embedded into an invertible fuzzy topological space. Further, characterizations and basic properties of completely invertible fuzzy topological spaces are derived. Again, the relationship between homogeneity and invertibility is established. This includes sufficient conditions for the invertibility of a homogeneous fuzzy topological space. The homogeneity of the inverting set as a subspace guarantees the homogeneity of the parent fuzzy topological space. In general, a completely invertible fuzzy topological space need not be homogeneous and a strongly homogeneous fuzzy topological space need not even be invertible. Finally, the orbits in invertible fuzzy topological spaces are studied and characterized. It is noted that in completely invertible fuzzy topological spaces, the orbits are always dense. On the other hand, if a completely invertible fuzzy topological space is not homogeneous then the orbits are neither open nor closed. Further, in a completely invertible fuzzy topological space, if any orbit as a subspace is non-trivial, then it is completely invertible. Consequently, every completely invertible fuzzy topological space is the disjoint union of homogeneous dense subspaces in which every non-trivial subspace is completely invertible. If a fuzzy topological space (X, F) is invertible, then there exists an inverting fuzzy set g and an inverting map θ of (X, F). This g and θ together are called an inverting pair of (X, F).

Chapter 4 closely examines the structure of these inverting pairs and observes that whenever θ is an inverting map so is θ^{-1}. Further, it is noted that if (g, θ) is an inverting pair then $\theta(g) = \theta^{-1}(g)$, whenever g and $\theta(g)$ are not quasi-coincident. Also, if (g, e) is an inverting pair, where e is the identity map, then the condition $\frac{1}{2} \leq g$ gives a clear picture about the structure of g. Based on the inverting maps, two types of invertible fuzzy topological spaces are introduced and their characterizing properties are derived. In the case of completely invertible fuzzy topological spaces, this classification actually produces some significant results. Further, the collection of all completely invertible finite fuzzy topological spaces is classified into two.

Chapter 5 investigates mainly the effect of invertibility on certain fuzzy topological properties. Exploring the relation between invertibility and separation axioms, it is proved that weakly quasi-separated, quasi-separated, fuzzy quasi T_0 and T_1 properties of certain subspaces can be transferred to the parent fuzzy topological space with the help of invertibility. The regular, normal, separated and fuzzy T_2 properties of a subspace determined by a crisp open subset accounts for the same property of a completely invertible fuzzy topological space. Further, the effect of invertibility on the axioms of countability is examined and certain sufficient conditions for an invertible fuzzy topological space to satisfy the first and second axioms of countability are obtained. If the inverting set as a subspace is separable, then the parent space is

also separable. The role of invertibility on compactness and connectedness of a fuzzy topological space is also explored. It is proved that a type 2 completely invertible compact fuzzy topological space is strongly compact. Also, the compactness, α-compactness and α^*-compactness properties of the closure of an inverting crisp set as a subspace are carried over to the parent fuzzy topological space. Further, in a completely invertible strongly compact fuzzy topological space, every open fuzzy set contains a compact fuzzy subset. Interestingly, a completely invertible fuzzy topological space containing an open fuzzy connected subset has at most two components.

Chapter 6 studies sums, subspaces and simple extensions of different types of invertible fuzzy topological spaces and examines whether they remain in the same type or not. Even though complete invertibility is not additive, invertibility is an additive property. It is also proved that being type 1 and type 2 are not hereditary properties. While a type 1 invertible fuzzy topological space remains type 1 under simple extensions, type 2 looses that nature under simple extensions. But, type 2 nature of a completely invertible fuzzy topological space is retained with simple extensions. A related problem is to investigate the invertibility of the associated spaces and exploration in that direction produced some significant results. Invertibility of the quotient spaces of an invertible fuzzy topological space is also examined. A thorough investigation on the invertibility of the product space of a family of invertible fuzzy topological spaces has been carried out focusing on different types of invertible fuzzy topological spaces.

Chapter 7 extends the concept of invertibility to L-topological spaces and obtains certain properties of invertible L-topological spaces. It has been proved that stratification preserves invertibility of an L-topological space. Further, certain properties of inverting pairs are investigated. We also introduce completely invertible L-topological spaces and pinpoint some of their characteristics. Finally, we introduce two different types of invertible L-topological spaces and study their properties in relation to sums, subspaces and simple extensions. We also investigate the relationship between invertibility and countability axioms in L-topological spaces. In this direction, we prove that first countable, second countable and separable properties of certain subspaces are transferable to the parent L-topological space with the help of invertibility. We further investigate the effect of invertibility on the separation axioms in L-topological spaces and obtain certain local to global properties of invertible L-topologies.

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